Cryptographic Primitives in Blockchain

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Outline

Introduction

What is a fingerprint? Is it useful? How?

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Goal: To generate fingerprints for digital data, e.g., a fingerprint for a text message or a video.

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Is it difficult to generate a fingerprint for digital data?

Cryptographic Hash Functions

What is a cryptographic hash function?

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  - A variable-length data block M

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- Output:
  - A fixed-size hash code $h = H(M)$

Cryptographic Hash Function - SHA256 - Example-1

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How to design a cryptographically secure hash function?

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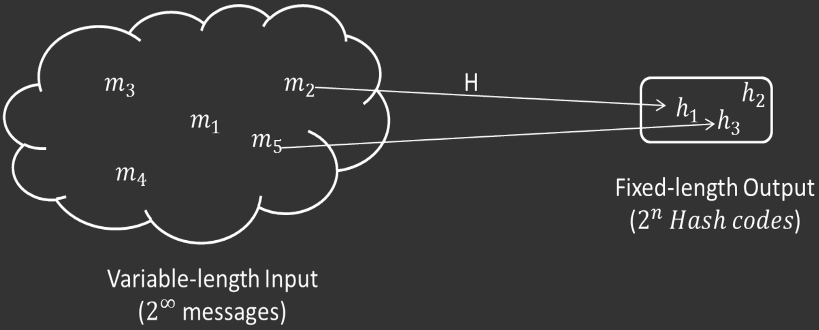

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- If the input is arbitrarily large, then the number of preimages per hash code is arbitrarily large. Hence, there will be collisions.
- Is it possible to design a collision resistant hash function?

One-way Property (Preimage Resistant)

For any given hash code h, it is computationally infeasible to find the input message m such that $H(m) = h$.

Second Preimage Resistant (Weak Collision Resistant)

For any given message m_1, it is computationally infeasible to find another message m_2 such that $m_1 \neq m_2$ and $H(m_1) = H(m_2)$.

Collision Resistant (Strong Collision Resistant)

It is computationally infeasible to find a pair of messages (m_1, m_2) such that $m_1 \neq m_2$ and $H(m_1) = H(m_2)$.

Security Attacks

- Two categories of attacks on hash functions.
  - Brute-force attacks - depend only on the bit length. (e.g., bit length of the hash code)
  - Cryptanalysis - depends on design flaw(s) of a particular hash function.

- Given the following entry of the "/etc/shadow" file (of Ubuntu 16.04), find the password of the user, "Alice" which contains [A-Za-z1-9] (Uppercase and/or lowercase letters and/or numbers).

```
Alice:$6$YIjqPaC8$ckYvhWkRkymmv/twBcANwa/L
WNjLsAdCHRToK3G9GIImPUWERnmFW2bUoOmLH
zUJ2tGr433QaOnHLdjDjc4Bs/:17648:0:99999:7:::
```

- Which property of the hash function you have to attack to find the password of Alice? Why?

Preimage Attack - A Brute-force Approach

For any given hash code h, it is computationally infeasible to find the input message m such that $H(m) = h$.

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- For an n-bit hash code, the level of effort is proportional to 2^n.

Second Preimage Attack - A Brute-force Approach

For any given message m_1, it is computationally infeasible to find another message m_2 such that $m_1 \neq m_2$ and $H(m_1) = H(m_2)$.

- Choose values of m_2 at random $(m_2 \neq m_1)$ and compute $H(m_2)$. Continue until a collision occurs, i.e., $H(m_2) = H(m_1)$.

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- For an n-bit hash code, the level of effort is proportional to 2^n.

What should be the length of the hash code h to prevent the preimage/second preimage attack?

Collision Resistant Attack - A Brute-force Approach

It is computationally infeasible to find a pair of messages (m_1, m_2) such that $m_1 \neq m_2$ and $H(m_1) = H(m_2)$.

- What is the level of effort required to perform the collision resistant attack, i.e., to find the pair of messages (m_1, m_2) such that $m_1 \neq m_2$ and $H(m_1) = H(m_2)$?

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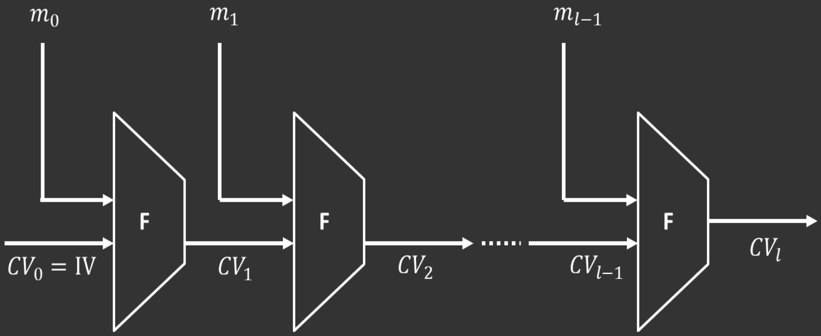
- For an n-bit hash code, the level of effort is **roughly** proportional to $2^{n/2}$.

What should be the length of the hash code h to prevent the preimage/second preimage/collision-resistant attack?

Merkle-Damgård Construction

- Most hash functions in use today follow the Merkle-Damgård structure.



- IV - Initial value
- m_i - i-th input block
- CV_i - Chaining variable
- CV_l - Hash code
- l - Number of input blocks
- F - Compression function

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Public-key Cryptography

What are the applications of public-key cryptography?

Is it difficult to distribute a key/secret between communicating parties?

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  - Public-key algorithms (RSA, ElGamal, etc.)
    - Mathematical functions
    - Two keys - one is public, the other is private/secret

Symmetric-key Cryptosystem - Encryption/Decryption

- m - Plaintext (Message)
- c - Ciphertext
- k - Secret-key shared between Alice and Bob

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- The need for public-key cryptosystems...

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  - Bobby Inman (Director of the NSA) - Discovered at NSA in the mid-1960s

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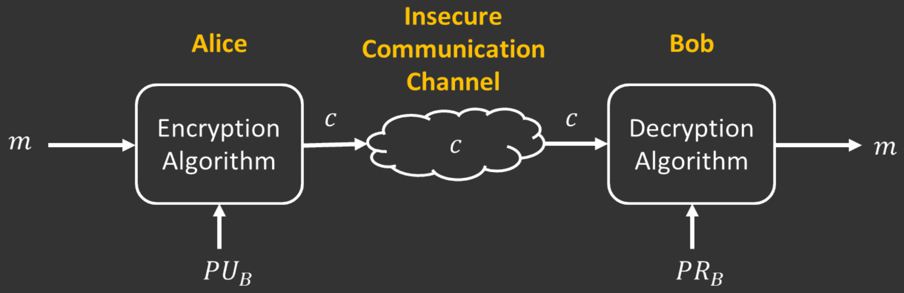

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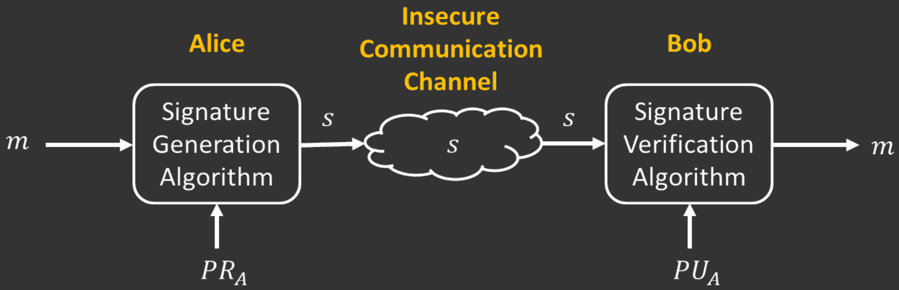
- c - Ciphertext
- PU_B - Bob's public-key
- PR_B - Bob's private-key

Public-key Cryptosystem - Digital Signature



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  - Key exchange - Uses the private key(s) of sender and/or recipient.

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- Decryption - should be computationally feasible given the ciphertext C and the private key PR_k.

$$M = D(PR_k, C) = D[PR_k, E(PU_k, M)]$$

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- Given the public key PU_k, it must be computationally infeasible to determine the private key, PR_k.
- Given the public key PU_k and a ciphertext C, it must be computationally infeasible to recover the original message M.

Is it difficult to design a public-key cryptosystem? If we have to design a public-key cryptosystem, what do we need?

Trapdoor one-way function

- Trap-door one-way function - Easy to compute in one direction and infeasible to compute in the other direction without trapdoor information.
- Trapdoor one-way function

$$
\begin{aligned}
C &= f_k(M) && \text{easy, if k and M are known} \\
M &= f_k^{-1}(C) && \text{easy, if k and C are known} \\
M &= f_k^{-1}(C) && \text{infeasible, if C is known but k is unknown}
\end{aligned}
$$

Here, "easy" means possible in polynomial time.

Public-key Cryptosystems - Attacks

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- Known/Chosen plaintext attacks to derive the private key.

Digital Signature

What is a signature? Is it useful? How?

Goal: To generate a signature for digital data, e.g., sign a text message or a video.

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Is it difficult to sign digital data?

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If two communicating parties do not trust each other, digital signature enables them to communicate with each other securely.

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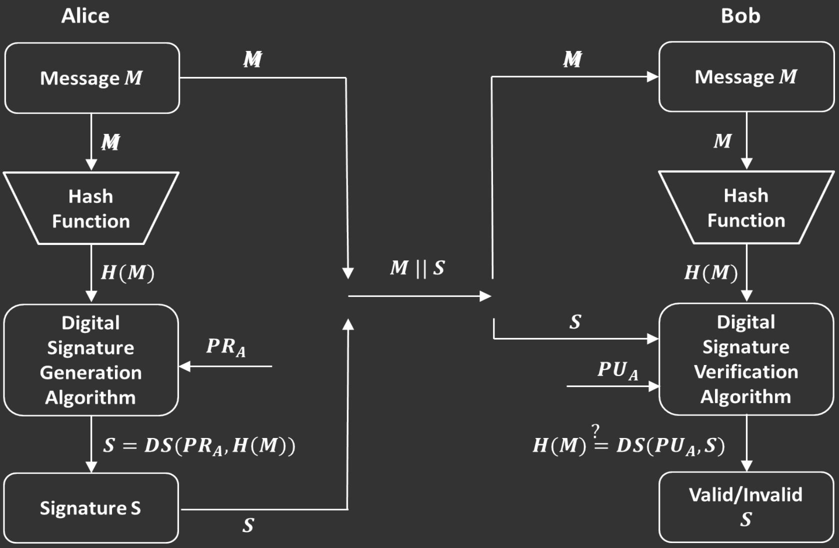

PR_A - Alice's Private Key PU_A - Alice's Public Key

Digital Signature - Algorithms

- Elgamal Digital Signature Scheme
- NIST Digital Signature Algorithm (DSA)
- Elliptic Curve Digital Signature Algorithm (ECDSA)
- Schnorr Digital Signature Scheme
- RSA-PSS Digital Signature Scheme

Number Theory

What is logarithm?

Logarithm

- The power to which a fixed number (base) must be raised to produce a given number.

$$a^x = b$$
$$x = \log_a b$$

Discrete Logarithm

$$a^x = b \text{ (mod } n)$$
$$x = \log_a b \text{ (mod } n)$$

- If n is a composite number, discrete log is not always possible.

Discrete Logarithm

$$a^x = b \ (\mathsf{mod}\ n)$$
$$x = \log_a b \ (\mathsf{mod}\ n)$$

- If n is a composite number, discrete log is not always possible.
- If n is a prime number, and a is a generator of the group, then discrete log exists.

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Given g, h, and a sufficiently large p (e.g., 1024-bit), finding the value of x is infeasible.

Discrete Logarithm Problem (DLP) - Example-1

Given g, x, and p, computing $h = g^x$ (mod p) is feasible.

- Let $p = 11$, $g = 2$, and $x = 5$, compute h.

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$$h = g^x \text{ (mod p)}$$
$$h = 2^5 \text{ (mod 11)}$$
$$h = 10$$

Discrete Logarithm Problem (DLP) - Example-2

Given g, h, and a sufficiently large p (e.g., 1024-bit), finding the value of x is infeasible.

$$h = g^x \text{ (mod p)}$$

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Discrete Logarithm Problem (DLP) - Example-2

Given g, h, and a sufficiently large p (e.g., 1024-bit), finding the value of x is infeasible.

$$h = g^x \text{ (mod p)}$$

- Let $p = 11$, $g = 2$, and $h = 10$, find x.

$10 \stackrel{?}{=} 2^1$ (mod 11) - No
$10 \stackrel{?}{=} 2^2$ (mod 11) - No
$10 \stackrel{?}{=} 2^3$ (mod 11) - No
$10 \stackrel{?}{=} 2^4$ (mod 11) - No
$10 \stackrel{?}{=} 2^5$ (mod 11) - Yes

There is no efficient way but to try all possible combinations until the answer is found. - A Discrete Logarithm Problem

Discrete Logarithm Problem (DLP) - Example-3

Given g, h, and a sufficiently large p (e.g., 1024-bit), finding the value of x is infeasible.

$$h = g^x \text{ (mod p)}$$

- Let $p = 131$, $g = 2$, and $h = 3$, find x.

Elgamal Digital Signature Scheme

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Is is feasible for an adversary to obtain the private key?

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The signature consists of the pair (S_1, S_2).

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  - Compute, $V_1 = \alpha^m \bmod p$.
  - Compute, $V_2 = (Y_A)^{S_1} \cdot (S_1)^{S_2} \bmod (p)$.

If $V_1 = V_2$, the signature is valid.

References

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Thank You.